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Go and return propagation of biphotons in fibre and polarization entanglement

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Abstract

Propagation of entangled photons in optical fibre is one of the fundamental issues for realizing quantum communication protocols. When entanglement in polarization is considered, a problem of compensating for the fibre effect on photons polarization arises. In this paper, we demonstrate an effective solution where a Faraday mirror allows us to cancel undesired effects of polarization drift in fibre. This technique is applied to a protocol for generating Bell states by a narrow temporal selection of the second-order intensity correlation function.

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1. Introduction

Transmission of photons in optical fibres is a fundamental tool both for realizing quantum communication protocols [1] and for experiments addressed to test foundations of quantum mechanics [2]. For example, long-distance quantum key distribution protocols in fibre have been realized by exploiting interferometric schemes [3] up to 100 km.

When quantum communication protocols are realized with photon states encoded in polarization some general difficulty must be considered connected with the transmission of photons through optical fibres. The photons propagating in fibres experience random polarization transformations due to the fluctuations of the fibre birefringence. These fluctuations are caused by mechanical, acoustic and thermal stresses in the fibre. Since it is impossible to completely isolate the fibre from the external disturbance, the problem of an effective compensation for the polarization drift becomes a crucial issue. For instance, a quantum key distribution (QKD) protocol with polarization-entangled states produced via a type-II parametric down-conversion (PDC) source was realized between a bank and the Vienna city hall by using a 1.45 km optical fibre [4]. Another example is a QKD network, also

involving fibre propagation of polarization-entangled photons, built among several research institutes in Boston [5]. In these examples, a correction for the fibre effect on polarization was introduced; moreover, this correction was tuned in time quite often due to the instability of the fibre. A similar correction has also been needed in experiments on the study of the interference structure in the second-order intensity correlation function [6, 7], visualized due to the spreading of the two-photon wave packet propagating through an optical fibre.

However, in experiments where the photon travels along the fibre in both directions (there and back), a more efficient scheme can be adopted. Indeed, in [8] it was shown that a Faraday cell followed by a mirror acts as a temporal inversion matrix, $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$, in the Jones formalism. Thus, the effect of the fibre on the polarization is reversed and cancels out when the photons are reflected back to the fibre by a Faraday mirror. Let us note that a similar transformation cannot be obtained with retardation plates and mirrors. An experiment with polarized light proved the viability of the scheme [9], which was later successfully applied to realize an interferometer [10]. In particular, such configuration can find a very useful application in ‘go and return’ quantum communication protocols as those of [11].

In this paper, we discuss the application of the Faraday mirror to observe quantum interference in the shape of the second-order correlation function where bipartite polarization-entangled states are produced in a type-II PDC process.

2. Theory

The parametric down-conversion process is a quantum effect without classical counterparts consisting of a spontaneous decay, in a nonlinear medium with non-zero second-order susceptibility, of one photon of a pump beam (usually generated by a laser) into a couple of strongly correlated photons, called biphotons.

Here, we investigate degenerate type-II PDC emission in the collinear direction to the pump beam, i.e. we choose an angle between the pump laser beam and the nonlinear crystal optic axis such that the frequency-degenerate pair of photons is emitted collinearly; furthermore, in type-II emission they have orthogonal polarizations. It must be mentioned that in collinear configuration the photons are not entangled in polarization at the exit of the crystal, but entanglement can be achieved by parting them on a beamsplitter (BS).

Due to the finite crystal size the biphotons acquire a certain spectral dispersion [13]. In low gain regime, neglecting higher orders, the state vector of the PDC light can be represented as a superposition of the vacuum state and the two-photon state given by an integral over the spectrum

$$|\Psi\rangle = |\text{vac}\rangle + \int d\Omega F(\Omega) [a_H^\dagger(\omega_0 + \Omega)a_V^\dagger(\omega_0 - \Omega) e^{i\Omega\tau_0} + a_V^\dagger(\omega_0 + \Omega)a_H^\dagger(\omega_0 - \Omega) e^{-i\Omega\tau_0}] |\text{vac}\rangle, \quad (1)$$

with $\omega_0 = \omega_p/2$, ω_p being the pump frequency, and a_H^\dagger and a_V^\dagger being the photon creation operators in the horizontal and vertical polarization modes (denoted by H , V). The phase factor $e^{\pm i\Omega\tau_0}$ is defined by the average temporal delay between orthogonally polarized photons, where $\tau_0 = DL/2$, $D \equiv 1/u_V - 1/u_H$ being the difference of the inverse group velocities and L the length of the crystal. The spectral and temporal properties of two-photon light are described by the two-photon spectral amplitude $F(\Omega)$ that establishes the natural bandwidth of PDC and for type-II phase matching has the shape

$$F(\Omega) = \frac{\sin(DL\Omega/2)}{DL\Omega/2}. \quad (2)$$

The second-order correlation function $G^{(2)}(\tau)$ is given by the square modulus of the Fourier transform of $F(\Omega)$, which is called the biphoton amplitude [12],

$$G^{(2)}(\tau) \propto |F(\tau)|^2 = \left| \int d\Omega F(\Omega) \exp(i\Omega\tau) \right|^2. \quad (3)$$

The utmost importance of the second-order correlation function in quantum optics derives from the fact that it is related to the coincidence rate $R_c \sim \int dt \int d\tau G^{(2)}(t, \tau)$ that can be easily measured experimentally and is the measured observable in a plethora of experiments ranging from Bell inequality tests to quantum information protocols (as teleportation, quantum swapping, entanglement-based QKD protocols, etc).

When the state (1) is transmitted through an optical fibre with group-velocity dispersion (GVD), as it has been shown in [14], a Fourier transformation is performed over the biphoton amplitude $F(\tau)$. Thus, it takes the shape of the two-photon spectral amplitude $F(\Omega)$,

$$\tilde{F}(\tau) = \frac{1}{\sqrt{4\pi ik''z}} \exp\left(\frac{i(\tau - k'z)^2}{4k''z}\right) F(\Omega) \Big|_{\Omega = \frac{\tau - k'z}{2k''z}}, \quad (4)$$

where k' , k'' are, respectively, the first and the second derivatives of the fibre dispersion law $k(\omega)$ at the frequency ω_0 and z is the length of the fibre. This effect has a clear analogue with the propagation of a short optical pulse in a dispersive medium and results in changing the shape of the pulse to the shape of its spectrum. As it was shown in [6], $G^{(2)}(\tau)$ shows an interference structure depending on the polarization selection performed over each photon of a pair. Namely, if after dividing the two photons on a 50/50 beamsplitter⁴, they are registered either with the same orientations of polarization filters set at 45° to initial basis or with the orthogonal orientations of the polarization filters set at 45° and -45° , $G^{(2)}(\tau)$ takes the form ($\tau_f \equiv 2k''z/\tau_0$ being the typical width of the correlation function after the fibre [15])

$$G_{\pm}^{(2)}(\tau) \sim \left| e^{\frac{i(\tau' - \tau_0)^2}{2\tau_0\tau_f}} \operatorname{sinc}\left(\frac{\tau' - \tau_0}{\tau_f}\right) \pm e^{\frac{i(\tau' + \tau_0)^2}{2\tau_0\tau_f}} \operatorname{sinc}\left(\frac{\tau' + \tau_0}{\tau_f}\right) \right|^2, \quad (5)$$

which, by taking into account $\tau_f \gg \tau_0$, reduce to

$$G_{+}^{(2)}(\tau) \sim \frac{\sin^2(\tau/\tau_f) \cos^2(\tau/\tau_f)}{(\tau/\tau_f)^2}, \quad G_{-}^{(2)}(\tau) \sim \frac{\sin^4(\tau/\tau_f)}{(\tau/\tau_f)^2}. \quad (6)$$

In equation (6) $G_{+}^{(2)}$ and $G_{-}^{(2)}$ correspond, respectively, to parallel (both at 45°) and orthogonal (one at 45° and another one at -45°) orientations of the polarization filters in the output ports of the beamsplitter.

Now let us focus on the problem of polarization drift in the fibre and its influence on the polarization entanglement. Upon passing the fibre, the initial polarization state of each photon (horizontal or vertical) is transformed from the equator of the Poincaré sphere to some arbitrary point on its surface [16]. This unitary transformation can be modelled as a polarization rotation by means of a retardation plate with an arbitrary optical thickness δ and orientation of the optical axis to initial basis α .⁵ The use of Jones' formalism [17] for the transformation of the creation operators leads to

$$\begin{pmatrix} a_H^\dagger \\ a_V^\dagger \end{pmatrix} = \begin{pmatrix} t & r \\ -r^* & t^* \end{pmatrix} \begin{pmatrix} a_H^\dagger \\ a_V^\dagger \end{pmatrix} \quad (7)$$

⁴ Here, we only consider the case when two photons of the pair go into different ports of the beamsplitter.

⁵ In the following calculations we neglect the chromatic dispersion of the fibre, i.e., the optical thickness δ is considered constant for all the spectrum of PDC.



Figure 1. Variation with $t = \tau/\tau_f$ of $G_+^{(2)}$ (first two lines) and $G_-^{(2)}$ (last two lines). For both $G_+^{(2)}$ and $G_-^{(2)}$, the first line is for α varying between 0 and $\pi/2$ with δ fixed at $\delta = \pi/6, \pi/4, \pi/3, \pi/2$. The second line corresponds to δ varying between 0 and $\pi/2$ with α fixed at $\alpha = \pi/6, \pi/4, \pi/3, \pi/2$.

where

$$t = \cos(\delta) + i \sin(\delta) \cos(2\alpha) \tag{8}$$

$$r = i \sin(\delta) \sin(2\alpha). \tag{9}$$

By substituting these expressions into (1) it can be shown that the shape of $G_+^{(2)}$ and $G_-^{(2)}$ changes from (6) to

$$\begin{aligned} G_+^{(2)}(\tau) &\sim [\cos^2 \delta(1 + \sin^2 \delta) + \sin^4 \delta \cos^2(4\alpha)] \frac{\sin^2(\tau/\tau_f) \cos^2(\tau/\tau_f)}{(\tau/\tau_f)^2}, \\ G_-^{(2)}(\tau) &\sim [\sin^2(4\alpha) \sin^4 \delta \cos^2(\tau/\tau_f) + \sin^2(\tau/\tau_f)] \frac{\sin^2(\tau/\tau_f)}{(\tau/\tau_f)^2}. \end{aligned} \tag{10}$$

The general dependence of $G_{\pm}^{(2)}$ on the parameters δ and α is shown in figure 1. From this one can prove the complexity of the variation of the interference pattern as a function of α and δ .

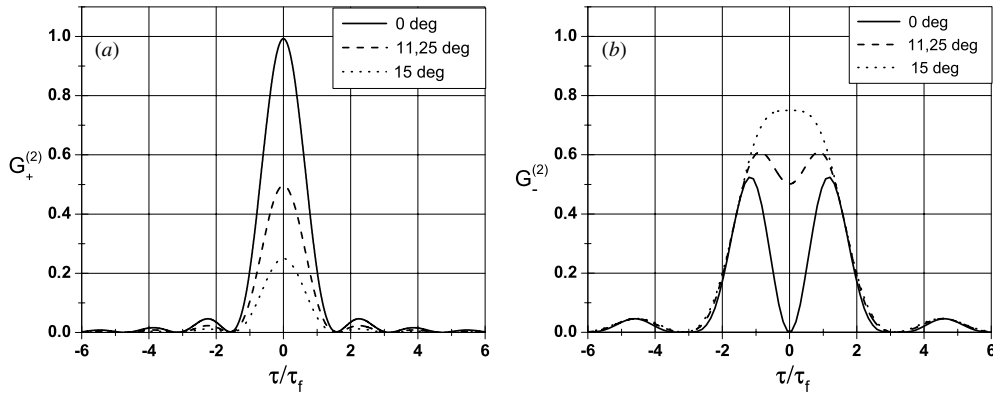


Figure 2. The second-order intensity correlation function for different orientations of the HWP acting on type-II PDC state in frequency-degenerate regime. $G_+^{(2)}$ corresponds to parallel orientations of polarization filters set to 45° (a); $G_-^{(2)}$ corresponds to orthogonal orientations of polarization filters set to 45° and -45° (b).

Let us now focus on the generation of Bell states. As it was shown in [6], a Bell state can be produced by performing a narrow temporal post-selection in the shape of $G^{(2)}(\tau)$. This can be realized, for instance, by analysing the $G^{(2)}(\tau)$ distribution with a Time-to-Amplitude Converter (TAC) and a multi-channel analyser (MCA), in which only specific channels are selected.

When one selects, after the beamsplitter, the central part of $G^{(2)}(\tau)$ corresponding to zero delay, i.e. $\tau = 0$ (which is equivalent to the selection of $\Omega = 0$ in the frequency domain), the state (1) becomes (see footnote 4) the polarization-spatial-entangled Bell state: $|\psi^+\rangle = \{a_{H1}^\dagger a_{V2}^\dagger + a_{V1}^\dagger a_{H2}^\dagger\}|\text{vac}\rangle$, where $a_{\sigma i}^\dagger$ are photon creation operators in the horizontal and vertical polarization modes (denoted by $\sigma = H, V$) and in two spatial modes of the beamsplitter (denoted by $i = 1, 2$). Being a pure maximally entangled state, $|\psi^+\rangle$ manifests 100% visibility of polarization interference⁶. However, if we consider the transformation of $|\psi^+\rangle$ in fibre, for instance modelled for the sake of simplicity by a half wave plate (HWP) with variable orientation α , then, as one can see from figure 2 (a section of figure 1), the visibility of polarization interference at $\tau = 0$ changes from 100% for the case when $\alpha = 0^\circ$ (which is equivalent to the absence of rotation) to zero when $\alpha = 11.25^\circ$. Moreover, at $\alpha = 22.5^\circ$ the two-photon state becomes an eigenstate of the measurement basis (photons of the pair are polarized at 45° and -45°) and the interference structure is completely erased. Thus, even considering the simplest model of polarization rotation produced by the fibre, this leads to spoiling the polarization interference and to erasing the polarization entanglement.

This problem has motivated our choice of applying a Faraday mirror to compensate for these effects. Moreover, taking into account a wide natural bandwidth of PDC spectra (typically of the order of ten nanometres) it is worth mentioning that the implementation of an appropriate Faraday mirror also allows us to correct for the effects related to frequency dependence of the effective birefringence in the fibre on the wavelength.

On the other hand, if the selected time interval corresponds to $\tau = \pm\pi\tau_f/2$, the relative phase between the two components of the state (1) becomes equal to π and the biphoton state represents the polarization–frequency singlet Bell state: $|\psi^-\rangle = \{a_{H1}^\dagger(\omega_1)a_{V2}^\dagger(\omega_2) -$

⁶ Here, we define the visibility in the traditional way as the difference of coincidences measured in orthogonal bases divided by their sum.

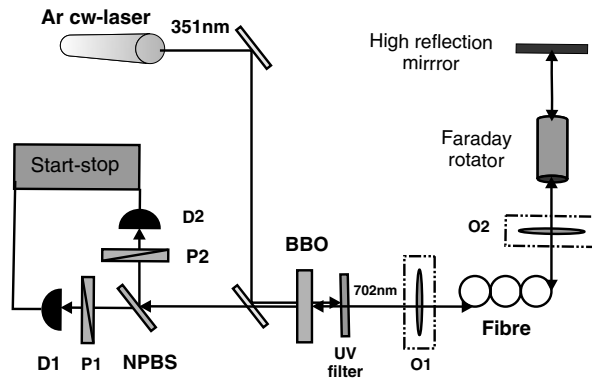


Figure 3. The experimental set-up. A cw Ar^+ laser at 351 nm pumps a type-II BBO crystal; O1, O2: microscope objectives; compensation for polarization drift in fibre is performed by free-space Faraday mirror, consisting of Faraday rotator and high-reflection mirror; NPBS: 50/50 non-polarizing beamsplitter; P1 and P2: Glan prisms; D1, D2: avalanche photodiodes. The output of the start-stop scheme is analysed by a multi-channel analyser (MCA).

$a_{V1}^\dagger(\omega_1)a_{H2}^\dagger(\omega_2)\}|\text{vac}\rangle$, where the two frequency modes are $\omega_1 = \omega_0 + \pi/2\tau_0$; $\omega_2 = \omega_0 - \pi/2\tau_0$. As one can see from (10), the visibility of polarization interference in this case remains 100% and does not depend on the polarization rotation induced by the fibre. This confirms a noticeable property of the singlet Bell state, namely, its invariance with respect to any polarization transformation. Therefore, for the case of the selection of $|\psi^-\rangle$ no compensation for the polarization drift is needed at all [18].

3. Experimental set-up and data

In our set-up, see figure 3, biphoton pairs were generated via spontaneous parametric down conversion by pumping a type-II 0.5 mm BBO crystal with a 0.5 Watt Ar^+ cw laser beam at a wavelength of 351 nm in the collinear frequency-degenerate regime. After the crystal, the pump laser beam was eliminated by a high-reflecting UV mirror and the PDC radiation was coupled into a 240 m long single-mode non-polarization maintaining fibre with field mode diameter of $4 \mu\text{m}$ by a $20\times$ microscope objective lens placed at the distance of 50 cm. This scheme provided imaging of the pump beam waist onto the fibre input with the magnification 1:40. For the efficient use of the pump beam, the pump was focused into the crystal using a UV lens with a focal length of 30 cm. The resulting beam waist diameter, $120 \mu\text{m}$, was small enough to be coupled with the fibre core diameter, but still sufficiently large not to influence the PDC angular spectrum.

After the first passage through the fibre, biphotons were addressed back by a free-space Faraday mirror, consisting of a Faraday rotator and a high-reflection mirror. In this way, the polarization effects of the fibre were compensated. Then, after having crossed the crystal, the photons passed the UV mirror needed for the injection of the pump beam into the crystal and reached the 50/50 beamsplitter preceding the detection apparatuses, consisting of polarizers and two avalanche photodiodes. The photocount pulses of the two detectors were sent to the START and STOP inputs of a TAC. The output of the TAC was finally addressed to an MCA, and the distribution of coincidences over the time interval between the photocounts of two detectors was observed at the MCA output.

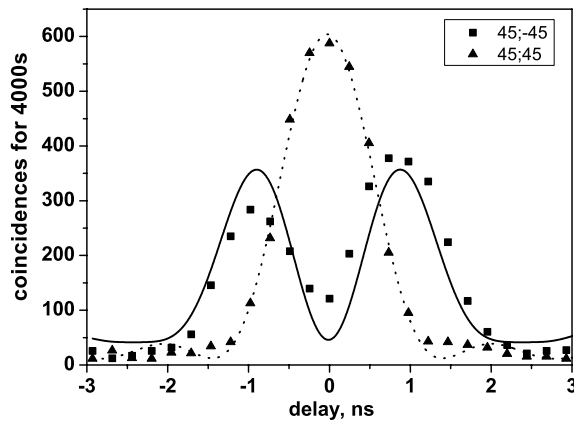


Figure 4. Experimental dependence of the coincidence count rate on the time delay between two photons for two cases: both polarizers are parallel and set at 45° (triangles) and polarizers are orthogonal and set at 45° and -45° (squares). The dashed and solid curves represent the fit of experimental data for $45^\circ; 45^\circ$ and $45^\circ; -45^\circ$ configurations, respectively.

The second-order intensity correlation function measured in the experiment for the case when both polarizers are parallel (set at 45°) and orthogonal (set at 45° and -45°) is presented in figure 4. The results show a good agreement with the theoretical predictions (6). Here, we would like to stress that the fibre was not placed in any isolation box or similar device so that it experienced the influence of heating in the lab, acoustic and mechanical stresses during all the acquisition time which exceeded 2 h. Moreover, the fibre we used was unwound from the spool and did not have an isolation plastic jacket. We have checked that in single-pass configuration under the same conditions the polarization state of light after the fibre drift on a time scale of few minutes (5–7).⁷

Thus, our results demonstrate that a perfect stability of the set-up in ‘go and return’ configuration over polarization fluctuations introduced by the fibre can be achieved by using a Faraday mirror.

The visibility of polarization interference at zero time delay measured in the experiment and calculated from figure 4 was 72% (after background subtraction). This reduction of the measured visibility with respect to theoretical predictions is explained by the length of the fibre, insufficient to make the spreading of $G^2(\tau)$ significantly larger than the time resolution of the set-up. Indeed, the fibre was chosen to be only 2×240 m long in order to avoid high losses of the signal (about 12 dB km^{-1} at 702 nm). A more detailed discussion on the influence of the fibre length on the visibility of polarization interference can be found in [7].

4. Conclusions

In conclusion, we demonstrated an application of the ‘go and return’ scheme for generating entangled states with the help of the spreading of the second-order intensity correlation function. We have shown that the implementation of the Faraday mirror allows us to compensate for the polarization drift in the fibre, which is crucial for obtaining polarization-

⁷ The strong coupling between fibre and environment and the polarization effects induced by this are the most important problems for quantum communication with polarization as qubit, on the other hand this coupling can be useful for studying properties of quantum channels as complete positivity [19] or decoherence control protocols.

entangled Bell states. Our experimental data verify the stability of the scheme under the influence of external disturbances over the fibre.

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